Root-Finding — Bisection Method

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1 Examples

Example-1: Use the bisection method to find the root of $f(x) = x^2 - 3$. Start with the interval [1, 2] and stop when $f(c) < 0.002$.

Solution

Table 1: Iterations for Example-1

a	b	$f(a)$	$f(b)$	$c = \frac{a+b}{2}$	$f(c)$	update
1	2	-2	1	1.5	-0.75	$a = c$
1.5	2	-0.75	1	1.75	0.0625	$b = c$
1.5	1.75	-0.75	0.0625	1.625	-0.35938	$a = c$
1.625	1.75	-0.35938	0.0625	1.6875	-0.15234	$a = c$
1.6875	1.75	-0.15234	0.0625	1.71875	-0.04590	$a = c$
1.71875	1.75	-0.04590	0.0625	1.73438	8.07398×10^{-3}	$b = c$
1.71875	1.73438	-0.04590	8.07398×10^{-3}	1.72657	-0.01896	$a = c$
1.72657	1.73438	-0.01896	8.07398×10^{-3}	1.73048	-5.43897×10^{-3}	$a = c$
1.73048	1.73438	-5.43897×10^{-3}	8.07398×10^{-3}	1.73243	1.31370×10^{-3}	$x^* = 1.73243$

Example-2: Use the bisection method to find the root of $f(x) = e^{-x}(3.2\sin x - 0.5\cos x)$. Start with the interval [3, 4] and stop when $f(c) < 0.001$.

Table 2: Iterations for Example-2

a	b	$f(a)$	$f(b)$	$c = \frac{a+b}{2}$	$f(c)$	update
3	4	0.04713	-0.03837	3.5	-0.01976	$b = c$
3	3.5	0.04713	-0.01976	3.25	5.84872×10^{-3}	$a = c$
3.25	3.5	5.84872×10^{-3}	-0.01976	3.375	-8.68108×10^{-3}	$b = c$
3.25	3.375	5.84872×10^{-3}	-8.68108×10^{-3}	3.3125	-1.87696×10^{-3}	$b = c$
3.25	3.3125	5.84872×10^{-3}	-1.87696×10^{-3}	3.28125	1.86703×10^{-3}	$a = c$
3.28125	3.3125	1.86703×10^{-3}	-1.87696×10^{-3}	3.296875	-3.42246×10^{-5}	$x^* = 3.296875$

Example-3: Use the bisection method to find the root of $f(x) = x^2 - \sin x - 0.5$. Start with the interval [0, 2] and stop when $f(c) < 0.001$.

Table 3: Iterations for Example-3

a	b	$f(a)$	$f(b)$	$c = \frac{a+b}{2}$	$f(c)$	update
0	2	-0.5	2.59070	1	-0.34147	$a = c$
1	2	-0.34147	2.59070	1.5	0.75251	$b = c$
1	1.5	-0.34147	0.75251	1.25	0.11352	$b = c$
1	1.25	-0.34147	0.11352	1.125	-0.13664	$a = c$
1.125	1.25	-0.13664	0.11352	1.1875	-0.01728	$a = c$
1.1875	1.25	-0.01728	0.11352	1.21875	0.04668	$b = c$
1.1875	1.21875	-0.01728	0.04668	1.203125	0.01434	$b = c$
1.1875	1.203125	-0.01728	0.01434	1.1953125	-0.00156	$a = c$
1.1953125	1.203125	-0.00155	0.01434	1.19921875	0.00637	$b = c$
1.1953125	1.19921875	-0.00155	0.00637	1.197265625	0.00240	$b = c$
1.1953125	1.197265625	-0.00155	0.00240	1.1962890625	4.19537×10^{-4}	$x^* = 1.1962890625$

2 Algorithm

Bisection Scheme Algorithm

Given a function $f(x)$ continuous on an interval $[a, b]$ and $f(a) \cdot f(b) < 0$
Do
    $c = {}^{(a+b)}/_{2}$
    if $f(a) \cdot f(c) < 0$ then $b = c$
    else $a = c$
while (none of the convergence criteria C1, C2 or C3 is satisfied)

where:

- C1. By testing if the total number of iterations equal to N.
- C2. By testing the condition $|c_i - c_{i-1}|$ (where i is the iteration number) less than some tolerance limit, say ϵ.
- C3. By testing the condition $|f(c_i)|$ less than some tolerance limit, say α (this condition is used throughout the examples given above).