Root-Finding — Secant Method

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1 Examples

Example-1: Use Secant method to find the root of the function $f(x) = \cos x + 2\sin x + x^2$ to 5 decimal places. Don't forget to adjust your calculator for "radians".

Solution

A closed form solution for x does not exist so we must use a numerical technique. The Secant method is given using the iterative equation:

$$x_{n+1} = x_n - f(x_n)\left[\frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})}\right], \tag{1}$$

We will use $x_0 = 0$ and $x_1 = -0.1$ as our initial approximations and substituting in (1), we have $x_{n+1} = -0.1 - 0.80533 * \left[\frac{-0.1}{0.80533-1}\right] = -0.51369$. The continued iterations can be computed as shown in Table 1 which shows a stop at iteration no. 5 since the error is $x_5 - x_4 < 10^{-5}$ resulting in a root of $x^* = -0.65926$, see Figure 1.

Table 1: Iterations for Example-1

Iteration no.	x_{n-1}	x_n	x_{n+1} using (1)	$f(x_{n+1})$	$x_{n+1} - x_n$
1	$x_0 = 0$	$x_1 = -0.1$	-0.51369	0.15203	-0.41369
2	-0.1	-0.51369	-0.60996	0.04605	-0.09627
3	-0.51369	-0.60996	-0.65179	6.60859×10^{-3}	-0.04183
4	-0.60996	-0.65179	-0.65880	4.08003×10^{-4}	-0.00701
5	-0.65179	-0.65880	-0.65926	5.28942×10^{-6}	$< 10^{-5}$

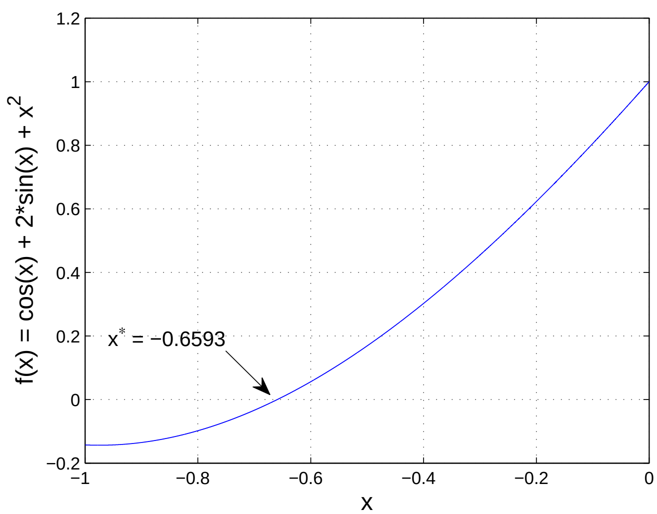

Figure 1: A plot of $f(x) = \cos x + 2\sin x + x^2$ using MATLAB.

Example-2: Use Secant method to find the root of the function $f(x) = x^3 - 4$ to 5 decimal places.

Solution

Since the Secant method is given using the iterative equation in (1). Starting with an initial value $x_0 = 1$ and $x_1 = 1.5$, using (1) we can compute $x_2 = 1.5 - (-0.625)\left[\frac{1.5-1}{-0.625-(-3)}\right] = 1.63158$. The continued iterations can be computed as shown in Table 2 which shows a stop at iteration no. 5 since the error is $x_5 - x_4 < 10^{-5}$ resulting in a root of $x^* = 1.58740$, see Figure 2.

Table 2: Iterations for Example-2

Iteration no.	x_{n-1}	x_n	x_{n+1} using (1)	$f(x_{n+1})$	$x_{n+1} - x_n$
1	$x_0 = 1$	$x_1 = 1.5$	1.63158	0.34335	0.13158
2	1.5	1.63158	1.58493	-0.01865	-0.04665
3	1.63158	1.58493	1.58733	-0.00054	0.0024
4	1.58493	1.58733	1.58740	-7.95238×10^{-6}	0.00007
5	1.58733	1.58740	1.58740	-7.95238×10^{-6}	$< 10^{-5}$

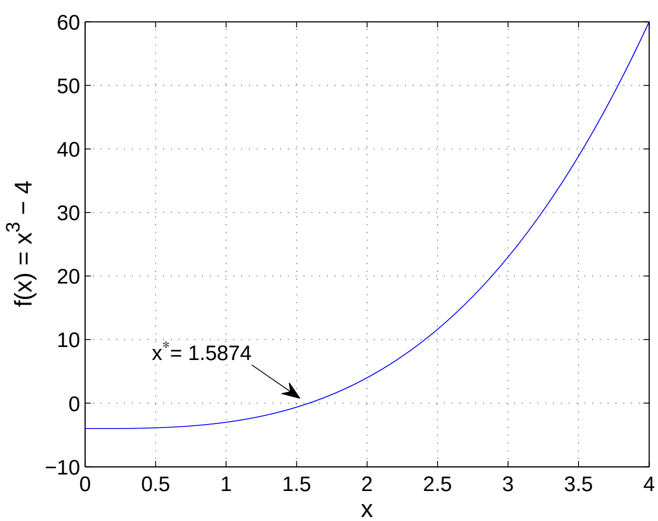

Figure 2: A plot of $f(x) = x^3 - 4$ using MATLAB.

Example-3: Use Secant method to find the root of the function $f(x) = 3x + \sin x - e^x$ to 5 decimal places. Use $x_0 = 0$ and $x_1 = 1$.

Solution

Using (1) we can compute $x_2 = 1-(1.12319)\left[\frac{1-0}{1.12319-(-1)}\right] = 0.47099$. The continued iterations can be computed as shown in Table 3 which shows a stop at iteration no. 6 since the error is $x_6 - x_5 < 10^{-5}$ resulting in a root of $x^* = 0.36042$, see Figure 3.

Table 3: Iterations for Example-3

Iteration no.	x_{n-1}	x_n	x_{n+1} using (1)	$f(x_{n+1})$	$x_{n+1} - x_n$
1	$x_0 = 0$	$x_1 = 1$	0.47099	0.26516	-0.52901
2	1	0.47099	0.30751	-0.13482	-0.16348
3	0.47099	0.30751	0.36261	5.47043×10^{-3}	0.0551
4	0.30751	0.36261	0.36046	9.58108×100^{-5}	-0.00215
5	0.36261	0.36046	0.36042	-4.26049×10^{-6}	-0.00004
6	0.36046	0.36042	0.36042	-4.26049×10^{-6}	$< 10^{-5}$

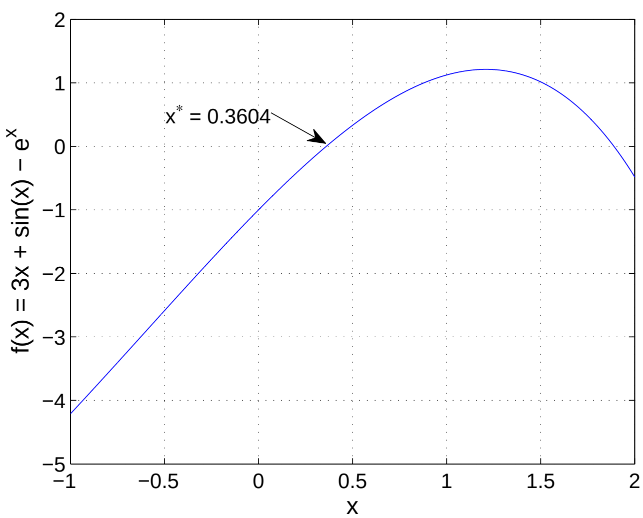

Figure 3: A plot of $f(x) = 3x + \sin x - e^x$ using MATLAB.

Example-4: Solve the equation $\exp(-x) = 3\log(x)$ to 5 decimal places using secant method, assuming initial guess $x_0 = 1$ and $x_1 = 2$.

Solution

Let $f(x) = \exp(-x) - 3\log(x)$, to solve the given, it is now equivalent to find the root of $f(x)$. Using (1) we can compute $x_2 = 2 - (-0.76775)\left[\frac{2-1}{-0.76775-(0.36788)}\right] = 1.32394$. The continued iterations can be computed as shown in Table 4 which shows a stop at iteration no. 5 since the error is $x_5 - x_4 < 10^{-5}$ resulting in a root of $x^* = 1.24682$, see Figure 4.

Table 4: Iterations for Example-4

Iteration no.	x_{n-1}	x_n	x_{n+1} using (1)	$f(x_{n+1})$	$x_{n+1} - x_n$
1	$x_0 = 1$	$x_1 = 2$	1.32394	-0.09952	-0.67606
2	2	1.32394	1.22325	0.03173	-0.10069
3	1.32394	1.22325	1.24759	-1.01955×10^{-3}	0.02434
4	1.22325	1.24759	1.24683	-7.27178×10^{-6}	-0.00076
5	1.24759	1.24683	1.24682	6.05199×10^{-6}	$< 10^{-5}$

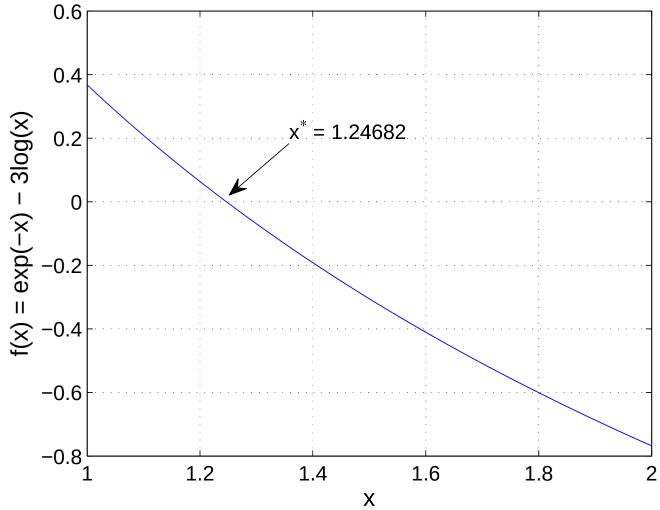

Figure 4: A plot of $f(x) = \exp(-x) - 3\log(x)$ using MATLAB.

2 Algorithm

Secant Method Algorithm

Given equation $f(x) = 0$, a predefined error ϵ, and a maximum no. of iterations N
Let the initial guesses be x_0 and x_1
Do

$$x_{n+1} = x_n - f(x_n)\left[\frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})}\right], \qquad n = 1, 2, \cdots$$

while the error $x_{n+1} - x_n < \epsilon$ or $n = N$

3 Exercises

1. Find the root of $x^2 = \frac{e^{-2x}-1}{x}$. $[x_0 = 1,\ x_1 = 2]$

2. Solve the equation $e^{\left(x^2-1\right)} + 10\sin 2x - 5 = 0$. $[x_0 = 0,\ x_1 = 1]$
3. Find the root of $f(x) = e^x - 3x^2$. $[x_0 = 0,\ x_1 = 1]$
4. Find the root of $f(x) = \tan x - x - 1$. $[x_0 = 0,\ x_1 = 1]$
5. Solve the equation $\sin 2x = \exp(x-1)$. $[x_0 = 0,\ x_1 = 1]$