Efficient Tomlinson-Harashima Precoding Ordering using QR Decomposition

Waleed Al-Hanafy and Stephan Weiss
w.alhanafy@eee.strath.ac.uk

CeSIP, EEE Dept., University of Strathclyde, UK

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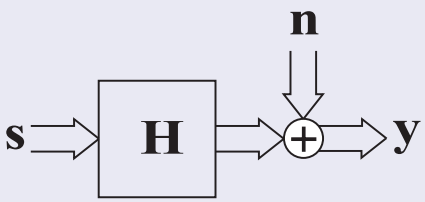
Overview

Preliminary: MIMO precoding/equalisation problem



- Linear precoding/equalisation methods
  - poor performance
  - less degree of freedom (DoF)
- Non-Linear methods: DFE, V-BLAST, THP
  - robust to interference & noise
  - higher DoF, ordering capability

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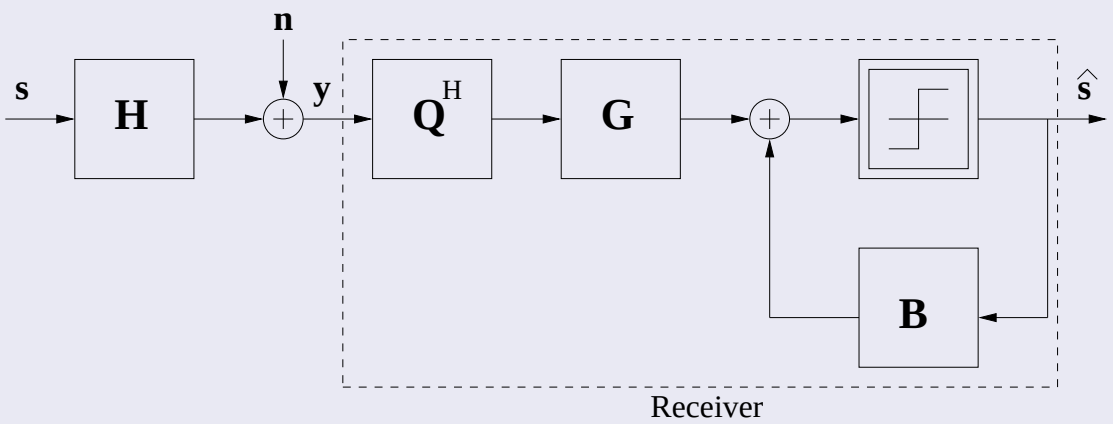
System model: DFE case via QR

MIMO DFE transceiver model



$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{n} \tag{1}$$

$$\mathbf{H} = \mathbf{Q}\mathbf{R} \tag{2}$$

$$\mathbf{R} = \mathbf{Q}^H\mathbf{H} = \begin{bmatrix} r_{11} & \cdots & r_{1N} \\ & \ddots & \vdots \\ 0 & & r_{NN} \end{bmatrix} \tag{3}$$

$$\mathbf{G} = diag\left[r_{11}^{-1}, r_{22}^{-1}, \cdots, r_{NN}^{-1}\right] \tag{4}$$

$$\mathbf{B} = \mathbf{I} - \mathbf{G}\mathbf{R} \tag{5}$$

QR interpretation algorithm for V-BLAST ordering

V-BLAST using QR (projection method) [1, 2]

Initialisation:

find k_N such that:

$$k_N = \arg \max_{1 \le k \le N} \left\| \left(\mathbf{I} - \mathbf{H}_k \mathbf{H}_k^{+} \right) \mathbf{h}_k \right\|^2$$

compute: $\alpha_N = \left(\mathbf{I} - \mathbf{H}_{k_N} \mathbf{H}_{k_N}^{+} \right) \mathbf{h}_{k_N}$

and then: $\mathbf{q}_N = \alpha_N / \left\| \alpha_N \right\|$

Recursion:

for $i = N-1, N-2, \cdots, 1$ find k_i such that:

$$k_i = \arg \max_{\substack{1 \le k \le N \\ k \neq k_{i+1}, \ldots, k_N}} \left\| \left(\mathbf{I} - \mathbf{H}_{k, k_{i+1}, \ldots, k_N} \mathbf{H}_{k, k_{i+1}, \ldots, k_N}^{+} \right) \mathbf{h}_k \right\|^2$$

compute: $\alpha_i = \left(\mathbf{I} - \mathbf{H}_{k_i, k_{i+1}, \ldots, k_N} \mathbf{H}_{k_i, k_{i+1}, \ldots, k_N}^{+} \right) \mathbf{h}_{k_i}$

and then: $\mathbf{q}_i = \alpha_i / \left\| \alpha_i \right\|$

compute: $\mathbf{Q} = [\mathbf{q}_1, \cdots, \mathbf{q}_N]$ and $\mathbf{R} = \mathbf{Q}^H \mathbf{H} \mathbf{P}_{VB}$.

QR efficient precoding ordering algorithm

QR using modified Gram–Schmidt process [3]

Initialisation:

$\mathbf{Q} = \mathbf{H}$, $\mathbf{R} = \mathbf{0}$, $\mathbf{p} = [1, 2, \cdots, M]$

Recursion:

for $i = 1, 2, \cdots, M$ find k_i such that:

$k_i = \arg \min_{i \le k \le M} \|\mathbf{q}_k\|^2$

interchange columns i and k_i in $\mathbf{Q}$, $\mathbf{R}$ and $\mathbf{p}$

$r_{ii} = \|\mathbf{q}_i\|$

$\mathbf{q}_i = \mathbf{q}_i / r_{ii}$

for $k = i + 1, \cdots, M$

$r_{ik} = \mathbf{q}_i^H \cdot \mathbf{q}_k$

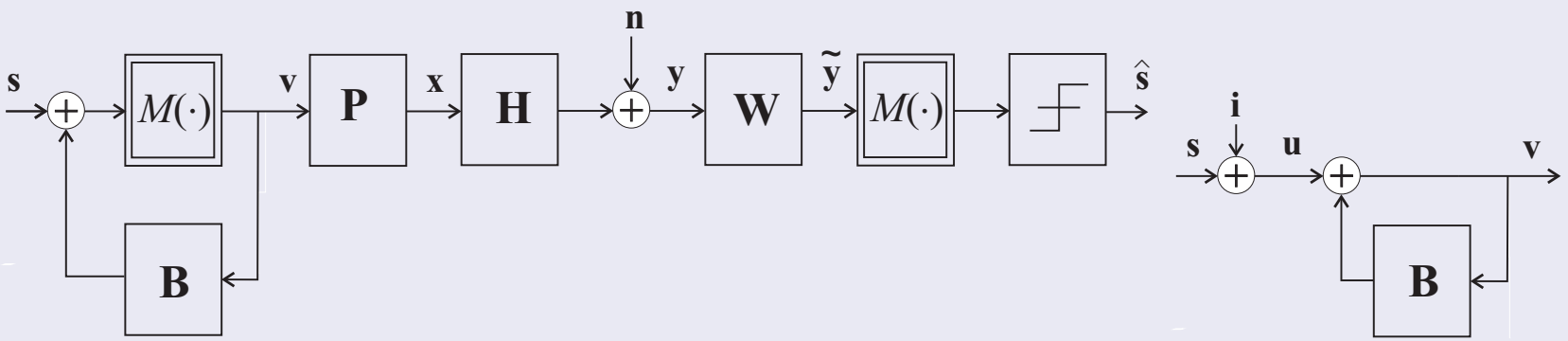
$\mathbf{q}_k = \mathbf{q}_k - r_{ik}\mathbf{q}_i$

end

end

System model: THP via QR

MIMO THP transceiver model [4]



$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} \tag{6}$$

$$\mathbf{Q}\mathbf{R} = \mathbf{H}\mathbf{P} \tag{7}$$

$$\tilde{\mathbf{y}} = \mathbf{W}\mathbf{H}\mathbf{P}(\mathbf{I}-\mathbf{B})^{-1}\mathbf{u} + \mathbf{W}\mathbf{n} \tag{8}$$

$$\begin{aligned} \mathbf{W} &= \mathbf{G}\mathbf{Q}^{H} \\ \mathbf{B} &= \mathbf{I} - \mathbf{G}\mathbf{R} \end{aligned} \tag{9}$$

Simulation results

A 5x5 MIMO THP system with QPSK modulation, 2000 channel realisations:

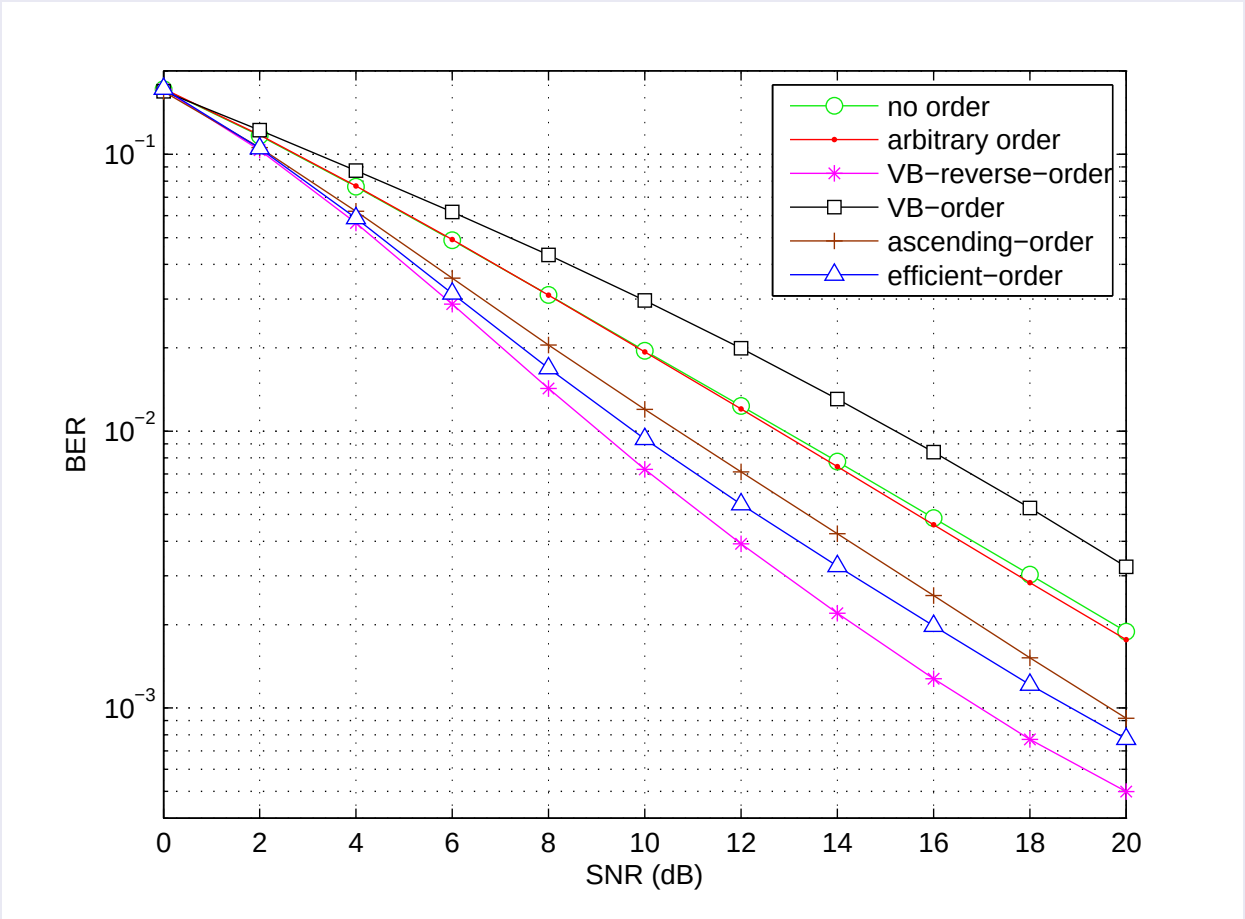
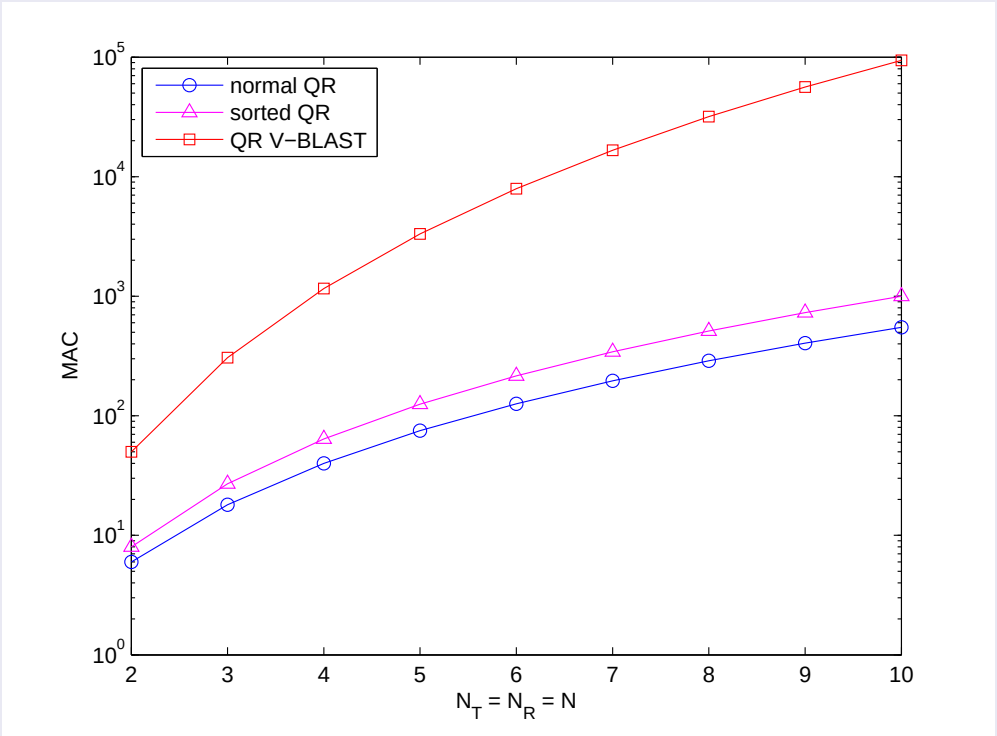


Figure: both "no order" and "arbitrary order" are chosen to signify sufficient channel realisations

Computation complexity comparison

Analysis and results:

normal QR	ordered QR	QR V-BLAST
$\frac{1}{2}\left(N^3+N^2\right)$	N^3	$\frac{5}{6}N^5+N^4+\frac{2}{3}N^3+\frac{1}{2}N^2$



Conclusions

- Efficient precoding ordering algorithm is preposed.
- Significant reduction in complexity is achieved compared to V-BLAST ordering.
- Precoding and detection ordering has to be reversed for efficient performance.
- BER improvement is achieved at the cost of slight computation complexity.

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G. Golden, C. Foschini, R. Valenzuela, and P. Wolniansky, "Detection Algorithm and Initial Laboratory Results Using V-Blast Space-Time Communication Architecture," *IEEE Electronics Letters*, vol. 35, no. 1, pp. 14–16, Jan. 1999.


J.-K. Zhang, A. Kavcic, and K. M. Wong, "Equal-Diagonal QR Decomposition and its Application to Precoder Design for Successive-Cancellation Detection," *IEEE Transactions on Information Theory*, vol. 51, no. 1, pp. 154–172, Jan. 2005.


D. Wubben, R. Bohnke, J. Rinas, V. Kuhn, and K. Kanimeyer, "Efficient Algorithm for Decoding Layered Space-Time Codes," *Electronics Letters*, vol. 37, pp. 1348–1350, Oct. 2001.


R. F. H. Fischer, C. Windpassinger, A. Lampe, and J. Huber, "Space-Time Transmission using Tomlinson-Harashima Precoding," in *4th International ITG Conference on Source and Channel Coding, Berlin*, Jan. 2002.

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