Chapter 3 — Methods of Analysis: 1) Nodal Analysis

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Overview

Reference:

[1] Alexander Sadiku, Fundamentals of Electric Circuits, 4th ed. McGraw-Hill, 2009.

Introduction

- Having understood the fundamental laws of circuit theory (Ohm's law and Kirchhoff's laws), we are now prepared to apply these laws to develop two powerful techniques for circuit analysis: *nodal analysis*, which is based on a systematic application of Kirchhoff's current law (KCL), and *mesh analysis*, which is based on a systematic application of Kirchhoff's voltage law (KVL).
- With the two techniques to be developed we can analyse almost any circuit by obtaining a set of simultaneous equations that are then solved to obtain the required values of current or voltage.
- Nodal analysis provides a general procedure for analysing circuits using node voltages as the circuit variables.
- In nodal analysis, we are interested in finding the node voltages.

Nodal Method Steps

Steps to Determine the Node Voltages:

1. Select a node as the reference node. Assign voltages v_1, $v_2, \cdots, v_{n-1}$ to the remaining $n - 1$ nodes. The voltages are referenced with respect to the reference node
2. Apply KCL to each of the $n - 1$ nonreference nodes. Use Ohm's law to express the branch currents in terms of node voltages
3. Solve the resulting simultaneous equations to obtain the unknown node voltages

Example-1

Calculate the node voltages in the circuit shown

- At node 1, applying KCL and Ohm's law gives

$$i_1 = i_2 + i_3$$

$$\text{or} \quad 5 = \frac{v_1 - v_2}{4} + \frac{v_1 - 0}{2}$$

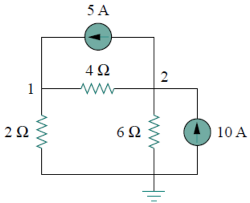
$$\Longrightarrow 3v_1 - v_2 = 20$$

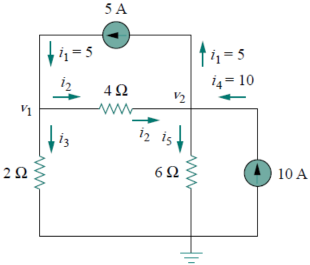
- At node 2, we do the same thing and get

$$i_2 + i_4 = i_1 + i_5$$

$$\text{or} \quad \frac{v_1 - v_2}{4} + 10 = 5 + \frac{v_2 - 0}{6}$$

$$\Longrightarrow -3v_1 + 5v_2 = 60$$





Example-1 (cont'd)

Solving both equations using either 1) *elimination technique*, or 2) *Cramer's rule*

We can get $v_1 = 13.33\,\mathrm{V}$ and $v_2 = 20\,\mathrm{V}$

Cramer's rule:

$$\begin{bmatrix} 3 & -1 \\ -3 & 5 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 60 \end{bmatrix}, \quad \Delta = \begin{vmatrix} 3 & -1 \\ -3 & 5 \end{vmatrix} = 15 - 3 = 12,$$

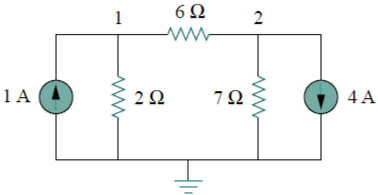
$$\Delta_1 = \begin{vmatrix} 20 & -1 \\ 60 & 5 \end{vmatrix} = 160, \quad \Delta_2 = \begin{vmatrix} 3 & 20 \\ -3 & 60 \end{vmatrix} = 240,$$

Thus $v_1 = \frac{\Delta_1}{\Delta} = 13.33\,\mathrm{V}$ and $v_2 = \frac{\Delta_2}{\Delta} = 20\,\mathrm{V}$, you can use these results to obtain $i_2, i_3, \text{and } i_5$

Exercise-1

Obtain the node voltages in the circuit in shown

Answer: $v_1 = -2$ V, $v_2 = -14$ V



Example-2

Determine the voltages at the nodes shown

At node1 $3 = i_1 + i_x \Rightarrow 3 = \frac{v_1 - v_3}{4} + \frac{v_1 - v_2}{2}$

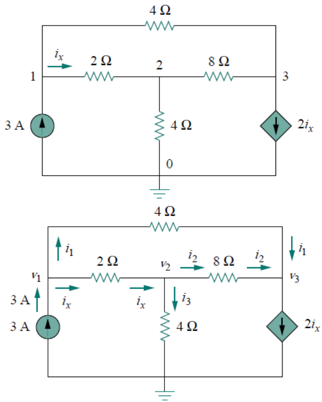
$\Longrightarrow 3v_1 - 2v_2 - v_3 = 12$

At node2 $i_x = i_2 + i_3 \Rightarrow \frac{v_1 - v_2}{2} = \frac{v_2 - v_3}{8} + \frac{v_2 - 0}{4}$

$\Longrightarrow -4v_1 + 7v_2 - v_3 = 0$

At node3 $i_1 + i_2 = 2i_x \Rightarrow \frac{v_1 - v_3}{4} + \frac{v_2 - v_3}{8} = \frac{2(v_1 - v_2)}{2}$

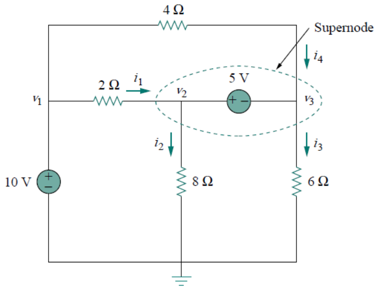
$\Longrightarrow 2v_1 - 3v_2 + v_3 = 0 \rightarrow v_1 = 4.8, v_2 = 2.4, v_3 = -2.4$

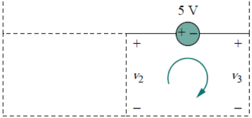


Supernode

A *supernode* is formed by enclosing a (dependent or independent) voltage source connected between two nonreference nodes and any elements connected in parallel with it.

- $v_1 = 10$ V
- $i_1 + i_4 = i_2 + i_3$
- $-v_2 + 5 + v_3 = 0$





Case 1

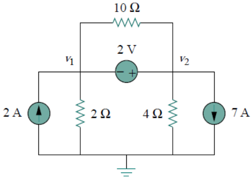
If a voltage source is connected between the reference node and a nonreference node, we simply set the voltage at the nonreference node equal to the voltage of the voltage source.

Case 2

If the voltage source (dependent or independent) is connected between two nonreference nodes, the two nonreference nodes form a generalised node or supernode; we apply both KCL and KVL to determine the node voltages.

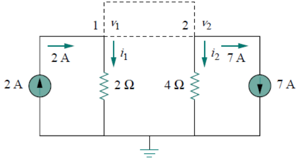
Example-3

For the circuit shown find the node voltages



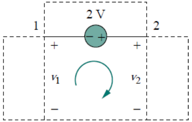
At supernode $2 = i_1 + i_2 + 7 \Rightarrow 2 = \dfrac{v_1 - 0}{2} + \dfrac{v_2 - 0}{4} + 7$

$$\Longrightarrow v_2 = -20 - 2v_1$$



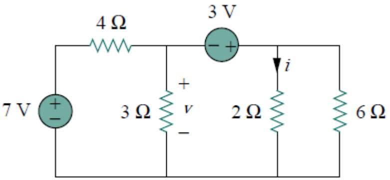
Apply KVL $-v_1 - 2 + v_2 = 0$

Thus $v_1 = -7.333$ V and $v_2 = -5.333$ V



Homework

Find v and i in the circuit shown using the nodal analysis method
Answer: -0.2 V, 1.4 A



Conclusion

Concluding remarks

- Nodal analysis method is studied as a key tool to analyse any circuit
- Basic nodal analysis steps is introduced highlighted by some examples
- The case of supernode is also given with examples
- Next — another analysis method known as mesh analysis will be introduced