Chapter 2 — Wye-Delta Transformations

Dr. Waleed Al-Hanafy
waleed_alhanafy@yahoo.com
Faculty of Electronic Engineering, Menoufia Univ., Egypt

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Electric Circuit Analysis I (ESE 233) — Lecture no. 3

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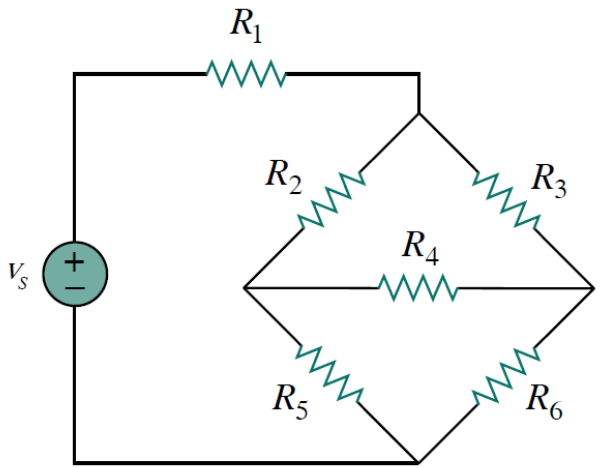
Overview

Reference:

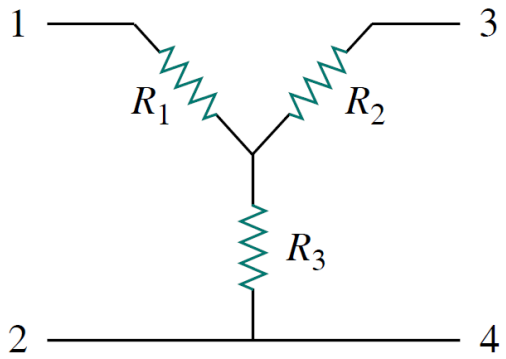
[1] Alexander Sadiku, Fundamentals of Electric Circuits, 4th ed. McGraw-Hill, 2009.

Problem Statement

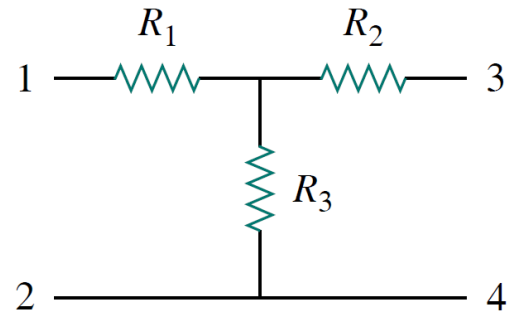
- Situations often arise in circuit analysis when the resistors are neither in parallel nor in series.
- For example, consider the bridge circuit shown.
- How do we combine resistors R_1 - R_6 when the resistors are neither in series nor in parallel?
- Many circuits of the type shown can be simplified by using three-terminal networks.
- These are the wye (Y) or tee (T) network, and the delta (Δ) or pi (Π) network as will be shown next.
- These networks occur by themselves or as part of a larger network.



Y, T, Δ, and Π Resistors Connections

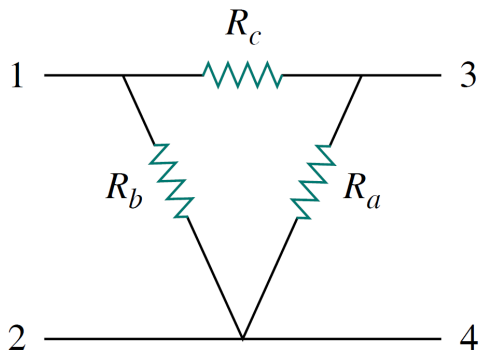


(a)

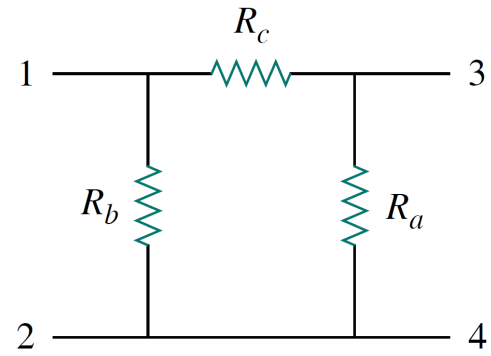


(b)

Two forms of the same network: (a) Y, (b) T.



(a)



(b)

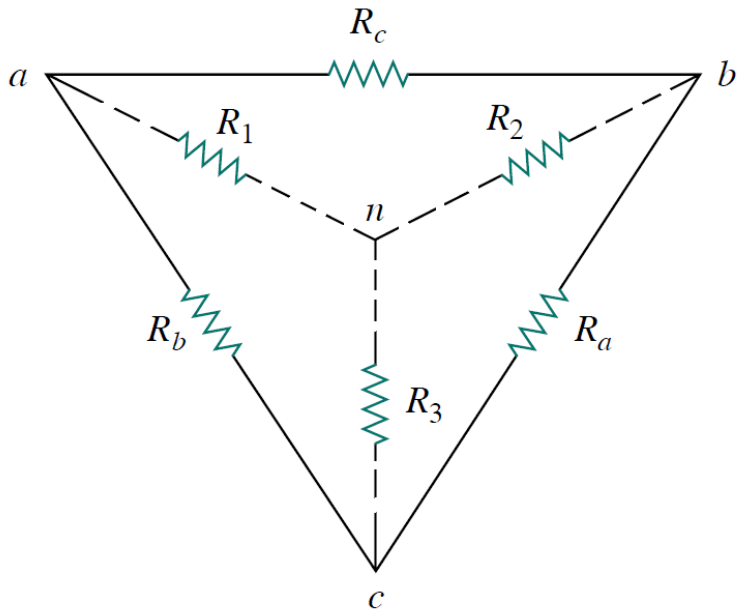
Two forms of the same network: (a) Δ, (b) Π.

Delta to Wye Conversion

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c} \qquad (1)$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c} \qquad (2)$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c} \qquad (3)$$

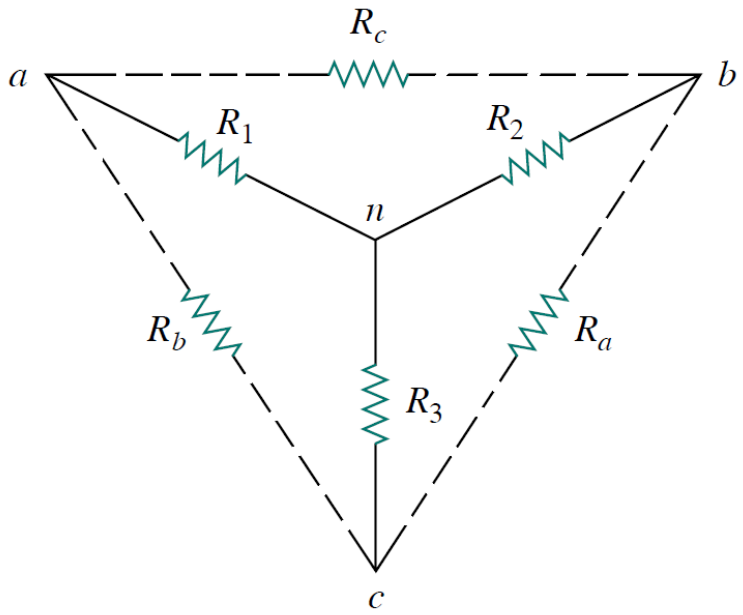


Wye to Delta Conversion

$$R_a = R_2 + R_3 + \frac{R_2 R_3}{R_1} \qquad (4)$$

$$R_b = R_1 + R_3 + \frac{R_1 R_3}{R_2} \qquad (5)$$

$$R_c = R_1 + R_2 + \frac{R_1 R_2}{R_3} \qquad (6)$$



Special Cases: Balanced Y and Δ

If in a Y resistors connection

$$R_1 = R_2 = R_3 = R_Y \tag{7}$$

and in a Δ resistors connection

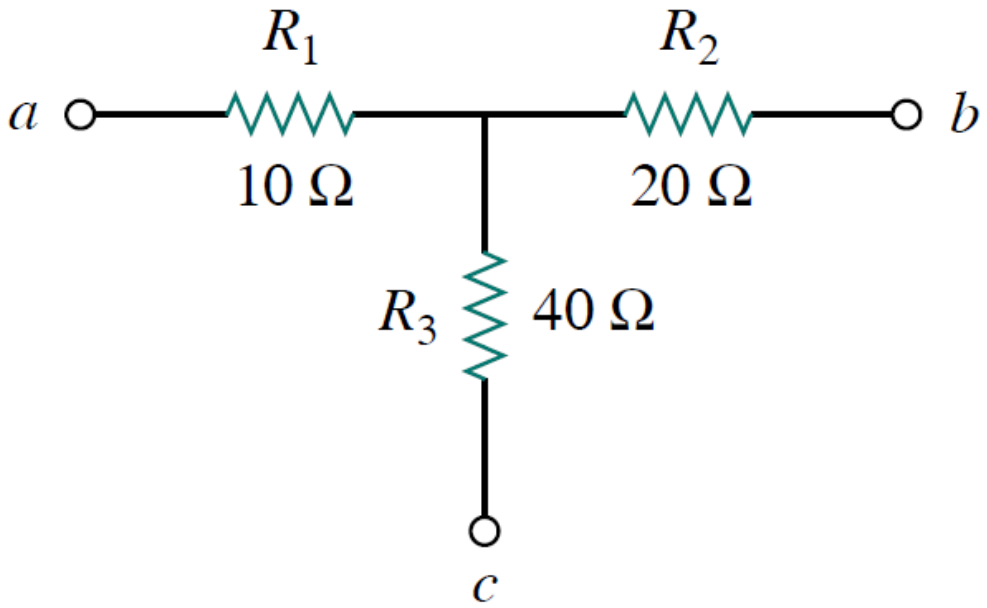
$$R_a = R_b = R_c = R_\Delta, \tag{8}$$

both Y and Δ networks are said to be balanced and in these cases:

$$R_Y = \frac{R_\Delta}{3} \qquad \text{or} \qquad R_\Delta = 3R_Y \tag{9}$$

Examples 1

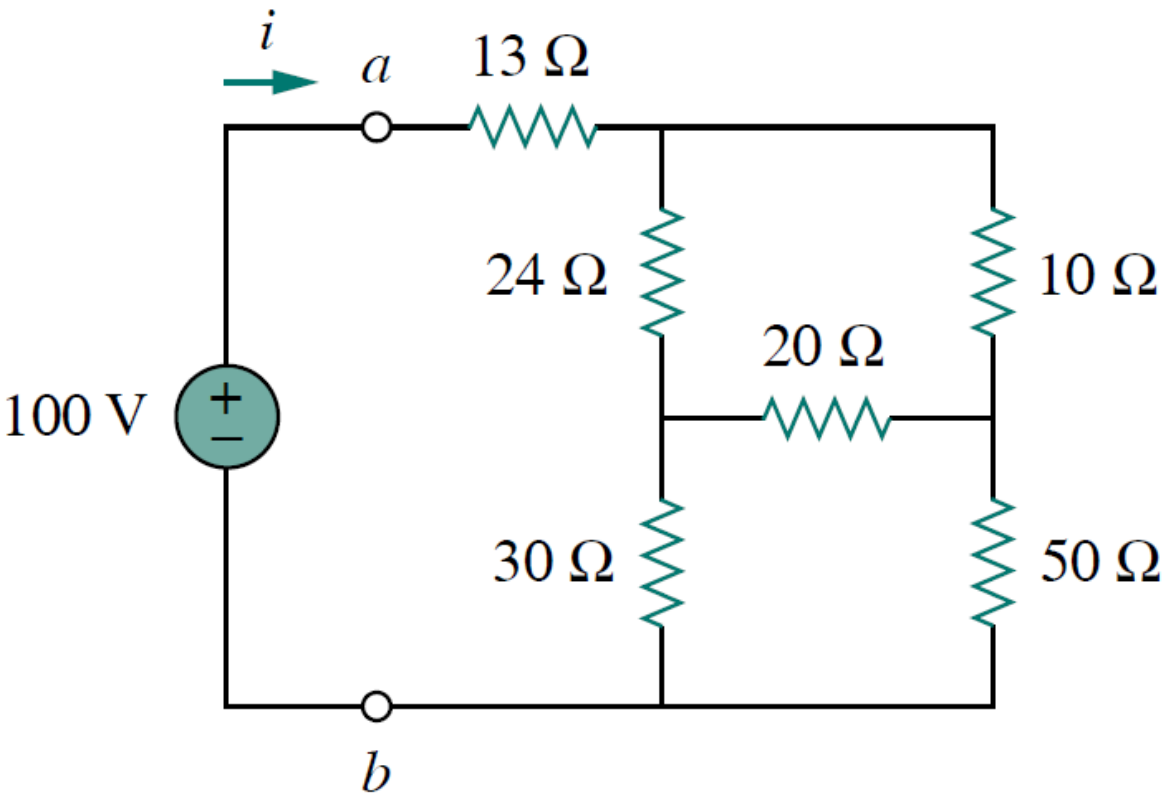
1) Transform the wye network shown to a delta network.
Answer: $R_a = 140\,\Omega,\ R_b = 70\,\Omega,\ R_c = 35\,\Omega$.



Examples 2

2) For the bridge network shown, find R_{ab} and i.

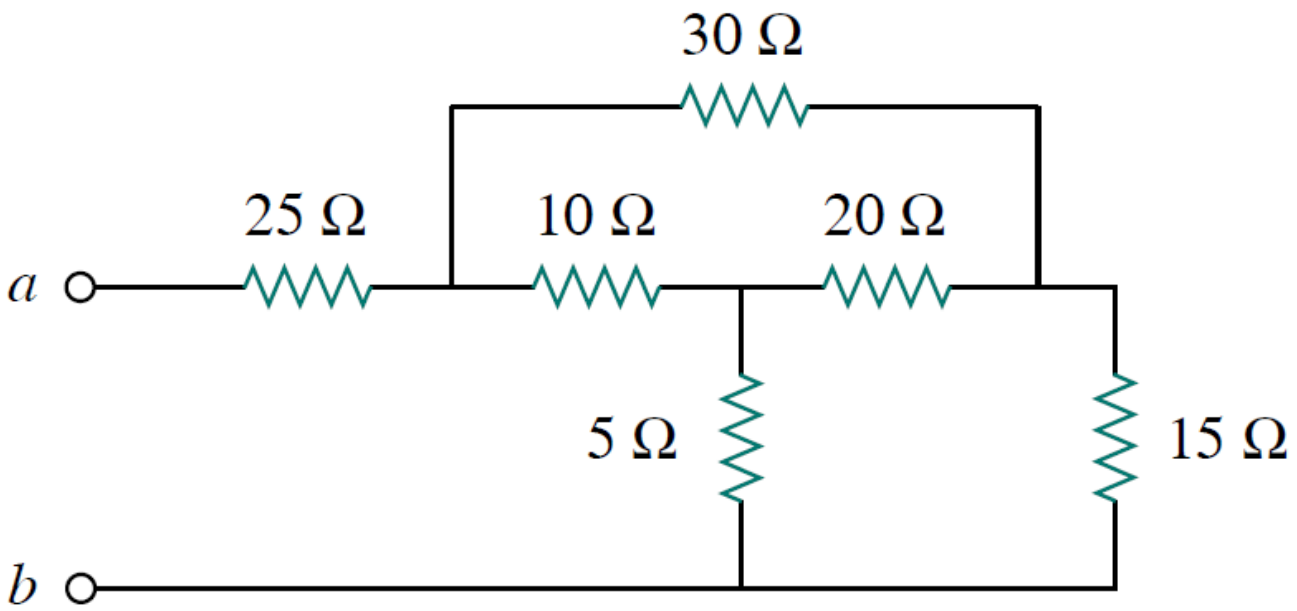
Answer: 40Ω, 2.5 A.



Examples 3

3) Obtain the equivalent resistance at the terminals a-b of the circuit shown.

Answer: 36.25Ω.



Conclusion

Concluding remarks

- In circuit analysis there are situations often arise when the resistors are neither in parallel nor in series.
- In these situations, it is very helpful to use three-terminal equivalent circuit transformations.
- These are known as Wye-Delta or Delta-Wye Transformations.
- Both cases are studied and discussed.
- Some illustrative examples are given.