MIMO Systems: Non-Linear Equalisation Techniques

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Overview

Maximum-likelihood detection (MLD)

- MLD is the optimum detection scheme as it optimally takes into account the properties of noise and interference [1]
- MLD receivers detect transmitted data symbols $\mathbf{s} \in \mathcal{S}^{N_t}$ on a per-vector basis by estimating the most probable transmitted data vector $\hat{\mathbf{s}}$ amongst all possible combinations $\mathcal{S}^{N_t}$ upon receiving $\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{v} \in \mathbb{C}^{N_r}$ such that

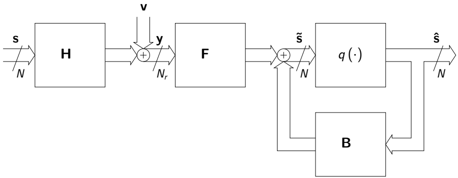
$$\hat{\mathbf{s}} = \underset{\mathbf{s} \in \mathcal{S}^{N_t}}{\operatorname{argmin}} \|\mathbf{y} - \mathbf{H}\mathbf{s}\|_2^2 , \tag{1}$$

- Obviously, MLD requires an exhaustive search of all possible M^{N_t} different data vectors to find the solution in (1) for every single vector detection
- This of course restricts the application of the MLD scheme for practical use as the complexity increases dramatically with the number of antennas N_t and the constellation size M
- In other words, MLD is not feasible for large orders of MIMO systems even with smaller modulation orders such as BPSK [2, 1]

Decision Feedback Equalisation (DFE)

- DFE is a simple and popular conventional non-linear technique that is established to separate data streams at the receiver of MIMO systems using successive interference cancellation
- This is shown to overcome the disadvantages of noise enhancement associated with linear equalisation [3] at the cost of extra complexity
- DFE was originally developed to combat ISI of SISO systems, i.e. temporal equalisation, (see e.g. [4, 5]) where previously detected symbols are used to assist in subsequent symbols detection
- Spatial DFE, in contrast, arises for MIMO system to deal with the multiuser interference
- A typical DFE system model is depicted in the figure below where the received data vector $\mathbf{y}$ is linearly processed by the feedforward filter matrix $\mathbf{F}$ before being considered by the non-linear feedback loop

DFE system model



- The undecided symbol $\tilde{\mathbf{s}}$ of the feedback loop is given by

$$\tilde{\mathbf{s}} = \mathbf{F}\mathbf{y} + \mathbf{B}\hat{\mathbf{s}} = \mathbf{F}\mathbf{H}\mathbf{s} + \mathbf{F}\mathbf{v} + \mathbf{B}\hat{\mathbf{s}}, \tag{2}$$

  where $\hat{\mathbf{s}}$ combines the decided estimates of $\mathbf{s}$ using the appropriate quantisation or decision device $q(\cdot)$
- For feasible realisability and to ensure spatial causality, the feedback filter matrix $\mathbf{B}$ must be of strictly triangular structure
- This can be achieved using the well-known QR factorisation of the channel matrix $\mathbf{H}$, that is $\mathbf{H} = \mathbf{Q}\mathbf{R}$ where $\mathbf{Q} \in \mathbb{C}^{N_r \times N_r}$ is a unitary matrix (i.e., $\mathbf{Q}\mathbf{Q}^{\mathrm{H}} = \mathbf{Q}^{\mathrm{H}}\mathbf{Q} = \mathbf{I}$) and $\mathbf{R} \in \mathbb{C}^{N_r \times N_t}$ is an upper triangular matrix

DFE analysis

- For simplicity, a MIMO system with equal numbers of transmit and receive antennas $N_t = N_r = N$ is considered
- Assuming perfect decisions of $\tilde{\mathbf{s}}$, i.e. $\hat{s}_i = q(\tilde{s}_i) = s_i, 1 \leq i \leq N$, which is a common assumption in DFE systems [6] called a "genie-aided" approach, the ZF solution can be formulated using (2) as

$$\begin{aligned} \mathbf{FH} + \mathbf{B} &= \mathbf{I} \\ \mathbf{FQR} + \mathbf{B} &= \mathbf{I} \end{aligned} \tag{3}$$

- Setting the feedforward filter $\mathbf{F} = \mathbf{G}\mathbf{Q}^{\mathrm{H}}$ in (3), results in the feedback filter $\mathbf{B} = \mathbf{I} - \mathbf{GR}$ where $\mathbf{G}$ is a diagonal matrix that is used to set the diagonal entries of $\mathbf{R}$ to unity, i.e.

$$\mathbf{G} = \mathrm{diag}\left(r_{11}^{-1}, \cdots, r_{NN}^{-1}\right) \tag{4}$$

DFE analysis (cont'd)

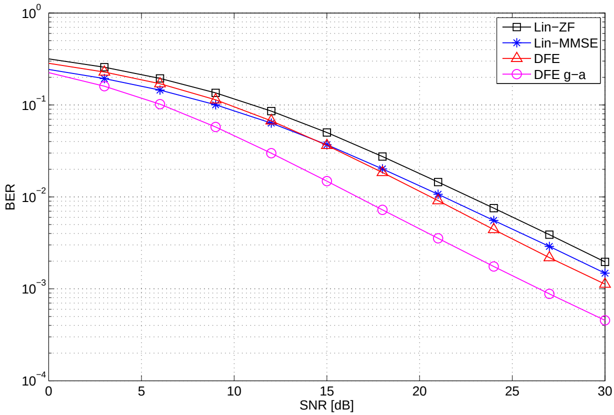
- With the absence of noise and by substituting these filter settings for **F** and **B** into (2) yields $\tilde{\mathbf{s}}$ as

$$\tilde{\mathbf{s}} = \begin{bmatrix} 1 & -b_{12} & \cdots & -b_{1N} \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & -b_{N-1N} \\ 0 & \cdots & 0 & 1 \end{bmatrix} \mathbf{s} + \begin{bmatrix} 0 & b_{12} & \cdots & b_{1N} \\ \vdots & \ddots & \ddots & \vdots \\ \vdots & & \ddots & b_{N-1N} \\ 0 & \cdots & \cdots & 0 \end{bmatrix} \hat{\mathbf{s}} \quad (5)$$

- Using (5), it is easy to note that the last-indexed symbol s_N of the transmitted vector $\mathbf{s}$ is an error-free symbol. This is utilised in detecting the spatially previous symbol s_{N-1} by subtracting out its interference signature on the statistics of $\tilde{s}_{N-1}$
- Proceeding further, both s_N and s_{N-1} are used to cancel their interference contributions from $\tilde{s}_{N-2}$ and so forth up to detecting the first-indexed symbol s_1

Results

A 4×4 MIMO system with 16-QAM transmission is considered. It is clearly noted that BER performance of the non-linear DFE system outperforms linear equalisation of ZF and MMSE, also the theoritical DFE genie-aided performance is shown for comparison



Conclusion

Concluding remarks

- Non-linear equalisation techniques are introduced for better performance than linear methods
- Both maximum-likelihood detection (MLD) and decision feedback equalisation (DFE) are given
- Simulation results comparisons are shown
- Next — another non-linear detection approach called V-BLAST will be studied

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