Chapter 2 — Basic Laws

Dr. Waleed Al-Hanafy
waleed_alhanafy@yahoo.com
Faculty of Electronic Engineering, Menoufia Univ., Egypt

MSA Summer Course:
Electric Circuit Analysis I (ESE 233) — Lecture no. 2

July 17, 2011

Overview

Reference:

[1] Alexander Sadiku, Fundamentals of Electric Circuits, 4th ed. McGraw-Hill, 2009.

Resistance and Resistivity

The physical property, or ability to resist current, is known as *resistance* and is represented by the symbol R. The resistance of any material with a uniform cross-sectional area A and length l is given as

$$R = \rho \frac{l}{A}, \qquad \text{ohms}\,(\Omega)$$

where ρ is known as the *resistivity* of the material in ohm-meters.

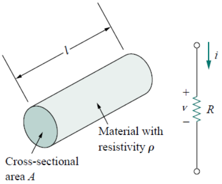


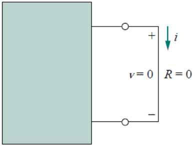
TABLE 2.1 Resistivities of common materials.

Material	Resistivity (Ω·m)	Usage
Silver	1.64×10^{-8}	Conductor
Copper	1.72×10^{-8}	Conductor
Aluminum	2.8×10^{-8}	Conductor
Gold	2.45×10^{-8}	Conductor
Carbon	4×10^{-5}	Semiconductor
Germanium	47×10^{-2}	Semiconductor
Silicon	6.4×10^{2}	Semiconductor
Paper	10^{10}	Insulator
Mica	5×10^{11}	Insulator
Glass	10^{12}	Insulator
Teflon	3×10^{12}	Insulator

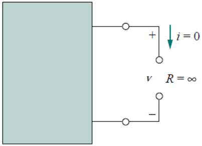
Ohm's Law

Ohm's law states that the voltage v across a resistor is directly proportional to the current i flowing through the resistor. *Georg Simon Ohm (1787–1854)*

$$\begin{aligned} v &\propto i \\ v &= iR \rightarrow R = \frac{v}{i} \quad 1\Omega = 1\mathrm{V/A} \end{aligned}$$



Since the value of R can range from zero to infinity, it is important that we consider the two extreme possible values of R. An element with $R = 0$ is called a *short circuit*. For a short circuit, $v = iR = 0$ showing that the voltage is zero but the current could be anything. Similarly, an element with $R = \infty$ is known as an *open circuit*, for an open circuit $i = v/R = 0$.



Conductance

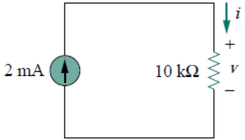
Definition: *Conductance is the ability of an element to conduct electric current; it is measured in mhos (℧) or Siemens (S).*

$$G = \frac{1}{R} = \frac{i}{v}, \qquad 1\text{S} = 1\mho = 1\text{A/V}$$

$$\begin{aligned} p &= i^2 R = \frac{v^2}{R} \\ &= v^2 G = \frac{i^2}{G} \end{aligned}$$

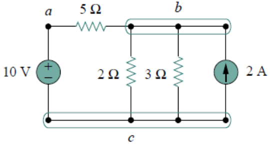
Example: For the circuit shown, calculate the voltage v, the conductance G, and the power p.

Answer: 20 V, 100 μS, 40 mW.



Definitions

- Branch: *A branch represents a single element such as a voltage source or a resistor*
- Node: *A node is the point of connection between two or more branches*
- Loop: *A loop is any closed path in a circuit*
- Two or more elements are in *series* if they are cascaded or connected sequentially and consequently carry the same current. Two or more elements are in *parallel* if they are connected to the same two nodes and consequently have the same voltage across them.



introduction

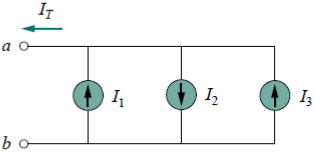
- Ohm's law by itself is not sufficient to analyse circuits. However, when it is coupled with Kirchhoff's two laws, we have a sufficient, powerful set of tools for analysing a large variety of electric circuits.
- Kirchhoff's laws were first introduced in 1847 by the German physicist *Gustav Robert Kirchhoff (1824–1887)*.
- These laws are formally known as Kirchhoff's current law (KCL) and Kirchhoff's voltage law (KVL).
    - Kirchhoff's first law (KCL) is based on the law of conservation of charge, which requires that the algebraic sum of charges within a system cannot change.
    - Kirchhoff's second law (KVL) is based on the principle of conservation of energy.

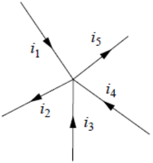
Kirchhoff's Current Law (KCL)

KCL: states that the algebraic sum of currents entering a node (or a closed boundary) is zero.

$$\sum_{n=1}^{N} i_n = 0,$$

where N is the number of branches connected to the node and in is the nth current entering (or leaving) the node.





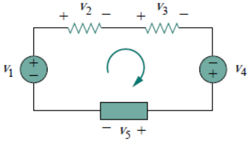
$$I_T = I_1 - I_2 + I_3$$

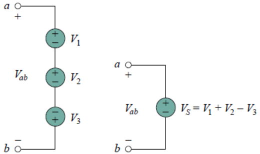
Kirchhoff's Voltage Law (KVL)

KVL: states that the algebraic sum of all voltages around a closed path (or loop) is zero.

$$\sum_{m=1}^{M} v_m = 0,$$

where M is the number of voltages in the loop (or the number of branches in the loop) and v_m is the mth voltage.
Example: $-v_1 + v_2 + v_3 - v_4 + v_5 = 0$, or $v_2 + v_3 + v_5 = v_1 + v_4$,
i.e., sum of voltage drops = sum of voltage rises





Series & Voltage Division

The *equivalent resistance* of any number of resistors connected in series is the sum of the individual resistances. For N resistors in series then,

$$R_{eq} = R_1 + R_2 + \cdots + R_N = \sum_{n=1}^{N} R_n$$

Notice that the source voltage v is divided among the resistors in direct proportion to their resistances; the larger the resistance, the larger the voltage drop. This is called the *principle of voltage division*, and the circuit in Fig. 2.29 is called a *voltage divider*.

$$v_n = \frac{R_n}{R_1 + R_2 + \cdots + R_N} v$$

Parallel & Current Division

The *equivalent resistance* of two parallel resistors is equal to the product of their resistances divided by their sum.

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N} \Longrightarrow R_{eq} = \frac{R}{N} \quad \text{if } R_1 = R_2 = \cdots = R_N = R$$

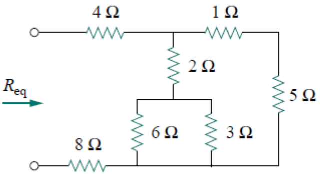
The *equivalent conductance* of resistors connected in parallel is the sum of their individual conductances.

$$G_{eq} = G_1 + G_2 + \cdots + G_N$$

$$i_1 = \frac{R_2 i}{R_1 + R_2}, \quad i_2 = \frac{R_1 i}{R_1 + R_2}$$

Example

Find R_{eq}
Answer 14.4Ω



Conclusion

Concluding remarks

- Basic circuits laws are studied
- These laws include, Ohm's law, Kirchhoff's two laws
- Series and parallel connections of resistors are discussed
- Voltage and current divisions are identified as a result of series and parallel resistors connection respectively
- Some illustrative examples are given.