MIMO Systems: Linear Equalisation & Precoding Techniques for Spatial Interference

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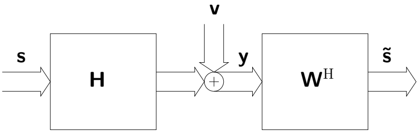
Adaptive DSP course for MSc & PhD students — Lecture no. 2

May 14, 2011

Overview

1. Equalisation
2. Precoding
3. Joint Transmit/Receive Processing
4. BER Performance
5. Conclusion and next lecture

System model



$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{v} \tag{1}$$

$\mathbf{y}$ — noisy N_r-dimensional received vector
$\mathbf{s}$ — N_t-dimensional transmit vector $\in \mathcal{S}^N$ is assumed to be a spatially-uncorrelated and uniformly distributed complex random vector process with zero-mean and variance σ_s^2 (i.e. $\mathbf{R}_{ss} = \mathbb{E}\left[\mathbf{s}\mathbf{s}^{\mathrm{H}}\right] = \sigma_s^2 \mathbf{I}_{N_t}$
$\mathbf{v}$ — noise vector with dimension $N_r \times 1$ drawn from $\mathcal{CN}\left(0, \sigma_v^2\right)$, or equivalently $\mathbf{R}_{vv} = \mathbb{E}\left[\mathbf{v}\mathbf{v}^{\mathrm{H}}\right] = \sigma_v^2 \mathbf{I}_{N_r}$
$\mathbf{H}$ — $N_r \times N_t$ flat-fading channel with entries h_{ij} are assumed i.i.d. complex Gaussian random variables with zero-mean and unit-variance $\mathbb{E}\left[|h_{ij}|^2\right] = 1$, i.e., $h_{ij} \in \mathcal{CN}(0, 1)$[1, 2]

Zero forcing (ZF) equaliser

The ZF approach is a very simple linear filter scheme that is accomplished by computing the Moore-Penrose pseudo-inverse of the channel matrix, $\mathbf{H}^{+}$. Accordingly, nulling in the ZF criterion is equivalent to completely cancelling the interference contributed by streams of other users (transmitting antennas) [3]. However, ZF receivers generally suffer from noise enhancement [4, 5]. The ZF filter is therefore given by

$$\mathbf{W}_{\mathrm{ZF}}^{\mathrm{H}} = \mathbf{H}^{+} = \left(\mathbf{H}^{\mathrm{H}}\mathbf{H}\right)^{-1}\mathbf{H}^{\mathrm{H}}, \tag{2}$$

and its output as

$$\tilde{\mathbf{s}}_{\mathrm{ZF}} = \mathbf{W}_{\mathrm{ZF}}^{\mathrm{H}}\mathbf{y} = \mathbf{s} + \left(\mathbf{H}^{\mathrm{H}}\mathbf{H}\right)^{-1}\mathbf{H}^{\mathrm{H}}\mathbf{v} \tag{3}$$

The error covariance matrix is therefore given by

$$\Phi_{\mathrm{ZF}} = \mathbb{E}\left[\left(\tilde{\mathbf{s}}_{\mathrm{ZF}} - \mathbf{s}\right)\left(\tilde{\mathbf{s}}_{\mathrm{ZF}} - \mathbf{s}\right)^{\mathrm{H}}\right] = \sigma_v^2\left(\mathbf{H}^{\mathrm{H}}\mathbf{H}\right)^{-1} \tag{4}$$

Noticeably, by referring to (4), it is evident that small eigenvalues of $\mathbf{H}^{\mathrm{H}}\mathbf{H}$ will lead to significant errors due to noise amplification. This, in fact, represents the main drawback of the ZF filter design as it disregards the noise term from the overall design and focuses only on perfectly removing the interference term from signal $\mathbf{s}$

Minimum mean square error (MMSE) equaliser

An improved performance over ZF can be achieved by considering the noise term in the design of the linear filter $\mathbf{W}^{\mathrm{H}}$. This is achieved by the MMSE equaliser, whereby the filter design accounts for a trade-off between noise amplification and interference suppression [3]. The MMSE filter is obtained by solving for error minimisation of the error criterion defined by

$$\varphi = \mathbb{E}\left[\mathbf{e}^{\mathrm{H}}\mathbf{e}\right] = \mathrm{tr}\left(\mathbb{E}\left[\mathbf{e}\mathbf{e}^{\mathrm{H}}\right]\right) \quad (5)$$

where the error vector $\mathbf{e} \stackrel{d}{=} \mathbf{s} - \tilde{\mathbf{s}} = \mathbf{s} - \mathbf{W}^{\mathrm{H}}\mathbf{y}$. Minimisation of φ leads to the Wiener-Hopf equation [6]

$$\mathbf{W}^{\mathrm{H}}_{\mathrm{MMSE}}\mathbf{R}_{yy} = \mathbf{R}_{sy} \quad (6)$$

This equation can also be obtained directly by invoking the orthogonality principle [7, 8] which states that the estimate $\tilde{\mathbf{s}}$ achieves minimum mean square error if the error sequence $\mathbf{e}$ is orthogonal to the observation $\mathbf{y}$, i.e., their cross-correlation matrix has to be the zero matrix $\mathbb{E}\left[\mathbf{e}\mathbf{y}^{\mathrm{H}}\right] = \mathbf{0}$. After some algebraic manipulation, the linear MMSE-sense filter is given by

$$\mathbf{W}^{\mathrm{H}}_{\mathrm{MMSE}} = \left(\mathbf{H}^{\mathrm{H}}\mathbf{H} + \frac{\sigma_v^2}{\sigma_s^2}\mathbf{I}_{N_t}\right)^{-1}\mathbf{H}^{\mathrm{H}} \quad (7)$$

MMSE equaliser (cont'd)

Similar to (4) the error covariance matrix using the MMSE filter in (7) is given as [3]

$$\Phi_{\mathrm{MMSE}} = \sigma_v^2 \left(\mathbf{H}^{\mathrm{H}} \mathbf{H} + \frac{\sigma_v^2}{\sigma_s^2} \mathbf{I}_{N_t} \right)^{-1} \quad (8)$$

Obviously by comparing (8) and (4), the error rate of the MMSE solution Φ_{MMSE} is less than its ZF counterpart Φ_{ZF} specifically at low SNR defined as

$$\mathrm{SNR} = \frac{\mathbb{E}\left[\|\mathbf{s}\|_2^2 \right]}{\mathbb{E}\left[\|\mathbf{v}\|_2^2 \right]} = \frac{\mathrm{tr}\left(\mathbf{R}_{ss} \right)}{\mathrm{tr}\left(\mathbf{R}_{vv} \right)} = \frac{\sigma_s^2 N_t}{\sigma_v^2 N_r} = P_{\mathrm{budget}} / \mathcal{N}_0 \quad (9)$$

where P_{budget} is the total transmit power budget and $\mathcal{N}_0$ is the total noise power at the receiver. At high SNR, the second term in (8) will vanish, which leads to asymptotic error performance similar to the ZF filter. Compared to the ZF filter in (2), the MMSE filter in (7) can be viewed as a "regularised" expression by a diagonal matrix of entries $\frac{\sigma_v^2}{\sigma_s^2}$, which is equal to the reciprocal of the SNR in (9) for equal numbers of transmit and receive antennas. This regularisation introduces a bias that gives a much more reliable result than (2) when the matrix is ill-conditioned, A matrix is said to be ill-conditioned if its condition number (the absolute ratio between the maximum and minimum eigenvalues) is too large., and/or the estimation of the channel is noisy.

Precoding

- It is worth noting that the detailed analysis presented above for both ZF and MMSE equalisation can also be derived in case of transmit processing [9, 10, 11], i.e. pre-equalisation (precoding).
- The transmitter in this case pre-distorts the transmit data in such a way that it cuts the interference seen at the receiver to a tolerable level.
- However, this is requires channel state information (CSI) knowledge at both sides which can be obtained at the transmitter for:
  - time division duplex assuming channel reciprocity
  - frequency division duplex using feedback channel.
- In addition, a careful design of the precoder has to consider a scaling factor to control the transmit power not to exceed P_{budget}

Singular value decomposition (SVD)

The task of interference cancellation can be jointly shared between transmitter and receiver. The SVD scheme can be formulated by factorising **H** as

$$\mathbf{H} = \mathbf{U}\mathbf{A}\mathbf{V}^{\mathrm{H}} \tag{10}$$

where $\mathbf{U} \in \mathbb{C}^{N_r \times N_r}$ and $\mathbf{V} \in \mathbb{C}^{N_t \times N_t}$ are unitary[1] matrices while $\mathbf{A} \in \mathbb{C}^{N_r \times N_t}$ is a diagonal matrix that contains the singular values of **H**, $a_i, i = 1, \cdots, N$ with

$$N = \min(N_t, N_r), \tag{11}$$

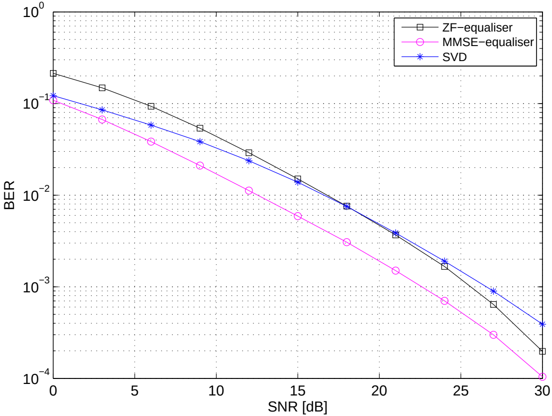
sorted in a descending order such that $a_1 \geq a_2 \geq \cdots$.
Now in order to decompose **H** into its singular values **A**, a precoder **V** and an equaliser $\mathbf{U}^{\mathrm{H}}$ can be jointly applied at the transmitter and at the receiver, respectively, such that the overall effective channel is given as

$$\mathbf{H}_{\mathrm{eff}} = \mathbf{U}^{\mathrm{H}}\mathbf{H}\mathbf{V} = \mathbf{U}^{\mathrm{H}}\mathbf{U}\mathbf{A}\mathbf{V}^{\mathrm{H}}\mathbf{V} = \mathbf{A} \tag{12}$$

thus decoupling the MIMO system

Results

A 4×4 MIMO system with QPSK transmission is considered. It is clearly noted that the MMSE filter outperforms its ZF counterpart and their BER performance converges at higher SNR where the regularisation factor $\sigma_v^2/\sigma_s^2 \ll 1$



Conclusion

Concluding remarks

- Linear equalisation and precoding systems for narrowband MIMO systems is studied
- Both ZF and MMSE equalisers are derived and analysed
- SVD scheme for joint precoding/equalisation is given
- Simulation results comparisons are shown
- Next — non-linear equalisation/detection of MIMO systems will be studied

- A. Lozano and C. Papadias, "Layered Space-Time Receivers for Frequency-Selective Wireless Channels," *IEEE Transactions on Communications*, vol. 50, no. 1, pp. 65–73, Jan. 2002.
- K. Kusume, M. Joham, W. Utschick, and G. Bauch, "Efficient Tomlinson-Harashima Precoding for Spatial Multiplexing on Flat MIMO Channel," in *IEEE International Conference on Communications*, vol. 3, May 2005, pp. 2021–2025.
- R. Bohnke, D. Wubben, V. Kuhn, and K.-D. Kammeyer, "Reduced Complexity MMSE Detection for BLAST Architectures," in *IEEE Global Telecommunications Conference, GLOBECOM '03*, vol. 4, Dec. 2003, pp. 2258–2262.
- A. Benjebbour, H. Murata, and S. Yoshida, "Comparison of Ordered Successive Receivers for Space-Time Transmission," in

IEEE VTS 54th Vehicular Technology Conference, VTC 2001 Fall, vol. 4, Atlantic City, NJ, USA, 2001, pp. 2053–2057.

C. Stierstorfer and R. F. H. Fischer, "Lattice-Reduction-Aided Tomlinson-Harashima Precoding for Point-to-Multipoint Transmission," *International Journal of Electronics and Communications, AEU*, vol. 60, no. 4, pp. 328–330, Apr. 2006.

J. Benesty, Y. A. Huang, and J. Chen, "A Fast Recursive Algorithm for Optimum Sequential Signal Detection in a BLAST System," *IEEE Transactions on Signal Processing*, vol. 51, no. 7, pp. 1722–1730, Jul. 2003.

R. F. H. Fischer, "MMSE DFE for high-rate MIMO transmission over channels with ISI," in *Fifth IEE International Conference on 3G Mobile Communication Technologies*, 2004.

J. Liu and W. A. Kizymien, "Improved Tomlinson-Harashima Precoding for the Downlink of Multi-User MIMO Systems," *Canadian Journal of Electrical and Computer Engineering*, vol. 32, no. 3, pp. 133–144, 2007.

- C. Peel, Q. Spencer, A. L. Swindlehurst, and B. Hochwald, "Downlink Transmit Beamforming in Multi-User MIMO Systems," in *IEEE Workshop on Sensor Array and Multichannel Signal Processing*, 2004, pp. 43–51.

- M. Joham, W. Utschick, and J. A. Nossek, "Linear Transmit Processing in MIMO Communications Systems," *IEEE Transactions on Signal Processing*, vol. 53, no. 8, pp. 2700–2712, Aug. 2005.

- R. F. H. Fischer and J. B. Huber, "Equalization Strategies for Transmission over Space and Time," *Electronik und Informationstechnik*, vol. 6, pp. 187–195, 2005.